A SCHEME FOR COUPLING A FLUID PRESHEATH SOLUTION TO A KINETIC SHEATH SOLUTION

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Abstract: A scheme for coupling ("matching") a quasineutral two-fluid (electron-ion) presheath solution to a non-neutral, collisionless kinetic sheath solution has been presented. The matching interface (the "sheath edge") is defined by the sheath-edge singularity of the presheath solution. A comparison of our model (characterized by cut-off Maxwellian distribution functions) with the "standard" one (assuming Boltzmann-distributed electrons) shows qualitative agreement but also some visible differences, possible reasons for which are discussed.

Key words: Bohm criterion; Debye sheath; plasma; plasma-wall transition; quasineutrality.

INTRODUCTION AND MODEL

The "Plasma-Wall Transition (PWT)" region, extending between the bulk plasma and the wall, consists of the presheath and sheath regions, which are dominated by different physical mechanisms [1] [2]. Clearly, the most satisfactory approach would be to simulate the whole plasma system in question self-consistently [3]. However, for most cases of interest this appears to be out of reach at present and so a possible first step is to start out from existing approximate presheath and sheath models and couple them by matching them in a reasonable manner. In this sense, we propose in this work a scheme for coupling a quasineutral two-fluid (electron-ion) presheath solution to a non-neutral, collisionless kinetic sheath solution for a *1d1v* case. In reality there is a smooth transition between the presheath and sheath regions, but our first-step model assumes a sharp presheath-sheath interface defined by the "sheath-edge singularity" of the presheath solution. This problem is of intrinsic interest, e.g., in fluid simulations, where the fluid equations breaks down at the sheath-edge singularity and appropriate boundary conditions can be obtained only from sufficiently realistic kinetic sheath models.

PRESHEATH QUANTITIES AT THE SHEATH EDGE

Let us assume the presheath to be described by a two-fluid solution, characterized at L_+ (the presheath side of the sheath edge) by the densities n_{ps}^e and n_{ps}^i , the fluid velocities u_{ps}^e and u_{ps}^i, and the temperatures T_{ps}^e and T_{ps}^i. These six parameters are interrelated by the quasineutrality condition and the "sheath singularity" condition,

$$n_{ps}^e = n_{ps}^i \quad \text{and} \quad u_{ps}^i = -c_{ps}^i, \qquad c_{ps}^i := \sqrt{\frac{k\left(T_{ps}^e + \gamma^i T_{ps}^i\right)}{m^i}}, \qquad (1)$$

with γ^i the ion polytropic coefficient. Other important quantities to be used are the electric current density and "sheath-edge plasma density"

$$J_{ps} = e\left(n_{ps}^i u_{ps}^i - n_{ps}^e u_{ps}^e\right), \qquad n_{ps} := n_{ps}^e. \qquad (2)$$

SHEATH MODEL

The potential $\Phi(x)$ is assumed to increase monotonically from $\Phi(0) = \Phi_0 < 0$ at the wall to $\Phi(L\text{-}) = 0$ at the sheath entrance. The electron distribution function for $0 < x < L$ is given by

$$f^e(x,v) = A^e \exp\left[-\frac{m^e v^2}{2kT_f^e} + \frac{e\Phi(x)}{kT_f^e}\right]\Theta\left(v_c^e(x) - v\right), \qquad (3)$$

where $v_c^e(x) = \sqrt{2e\left[\Phi(x) - \Phi_0\right]/m^e}$ is the electron "cut-off" velocity. Assuming the ions to enter the sheath region with a cut-off shifted Maxwellian distribution, we find

$$f^i(x,v) = A^i \exp\left[-\frac{1}{\left(v_{tf}^i\right)^2}\left\{-\sqrt{v^2 + \frac{2e\Phi(x)}{m^i}} - v_{mL}^i\right\}^2\right]\Theta\left(v_c^i(x) - v\right), \qquad (4)$$

where $v_c^i(x) = -\sqrt{\left(v_{cL}^i\right)^2 - 2e\Phi(x)/m^i}$ is the ion "cut-off" velocity and v_{mL}^i is the ion "Maxwellian-maximum" velocity at L-. In these equations, T_f^s is the "formal" temperature of species *s (s = e, i)*, with $v_{tf}^s \equiv \sqrt{2kT_f^s/m^s}$. At L-, the sheath parameters are interrelated by the quasineutrality condition

$$n_L^e = n_L^i \qquad (5)$$

and the "marginal kinetic Bohm criterion"

$$\int_{-\infty}^{v_{cL}^i} \frac{dv}{v^2} \exp\left[-\frac{m^i\left(v - v_{mL}^i\right)^2}{2kT_f^i}\right] = \frac{m^i}{A^i}\frac{n_L^e}{kT_f^e}\left[1 + \sqrt{\frac{-kT_f^e}{e\pi\Phi_0}}\frac{D^e}{C^e}\right], \qquad (6)$$

where $C^e := 1 + erf\sqrt{-e\Phi_0/kT_f^e}$ and $D^e := \exp\left[e\Phi_0/kT_f^e\right]$. The Bohm criterion follows from the requirement $\left[d\Phi(x)/dx\right]_{x\to L-} \le 0$, which ensures a non-oscillatory sheath potential at the sheath edge, and from the observation that for realistic situations the equality sign must hold [2] [4].

PRESHEATH-SHEATH COUPLING

We require the sheath-side particle densities n_L^s, fluid velocities u_L^s, and effective temperatures $T_{eff,L}^s$ to equal their presheath-side counterparts n_{ps}^s, u_{ps}^s and T_{ps}^s, respectively. Our presheath-sheath transition problem then involves sixteen parameters, which are interrelated by twelve equations. Hence, choosing any four parameters (in a consistent manner, though) will fix the remaining twelve. If the necessary four parameters are chosen from the presheath/sheath side, the corresponding sheath/presheath parameters are obtained by solving those equations.

Fortunately, we are not forced to solve all the original presheath-sheath transition equations simultaneously. It rather turns out that we can, by judicious eliminations of parameters, derive from them an "irreducible" set of equations for a significantly lower number of parameters. These must be solved first, and the other parameters can then be calculated successively from the remaining equations. For example, for given n_{ps}, J_{ps}, T_{ps}^e and T_{ps}^i an equation for the normalized potential can be obtained as

$$\frac{D^e(\tilde{\Phi}_f)}{C^e(\tilde{\Phi}_f)}\sqrt{\frac{\tilde{T}_f^e(\tilde{\Phi}_f)}{\pi}} = \sqrt{\mu}\left(\tilde{J}_{ps} + \sqrt{\frac{1+\gamma^i \tilde{T}_{ps}^i}{2}}\right) \qquad (7)$$

where $\mu = m^e/m^i$, $\tilde{T}^s = T^s/T_{ps}^e$, $\tilde{\Phi}_f = e\Phi_0/kT_f^e$ and $\tilde{J}_{ps} = J_{ps}/\left(en_{ps}\sqrt{2kT_{ps}^e/m^i}\right)$. This equation can be solved for $\tilde{\Phi}_f$. All other parameters can be then calculated successively from the remaining dimensionless equations [5].

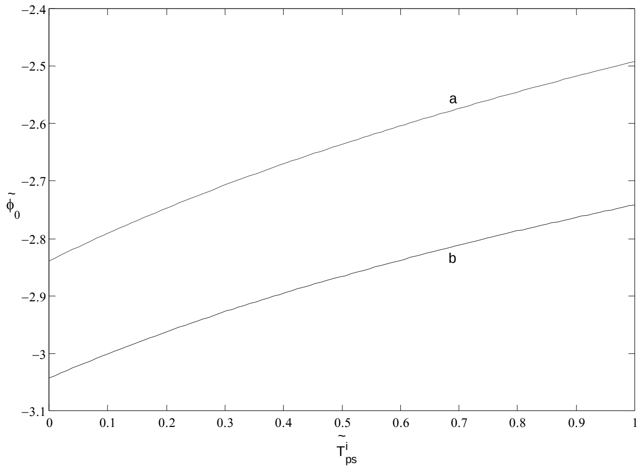

Fig. 1: Dependence of the normalized wall potential on normalized ion temperature - **(a)** according to Wesson [4] and **(b)** as obtained from our calculation.

CONCLUSIONS AND PERSPECTIVES

The main difference of our sheath model as compared with previous ones is that our sheath electrons have a cut-off Maxwellian distribution [Eq. (3)], leading to the density distribution

$$n^e(x) = n_L^e \exp\left(\frac{e\Phi(x)}{kT_f^e}\right)\frac{1+erf\sqrt{e(\Phi-\Phi_0)/kT_f^e}}{1+erf\sqrt{e(\Phi_L-\Phi_0)/kT_f^e}} \qquad (8)$$

whereas all other ("standard") sheath models known assume a Boltzmann distribution, $n^e(x) = n_L^e \exp\left(e\Phi(x)/kT_{ps}^e\right)$. A comparative plot of $\tilde{\Phi}_0 \equiv e\Phi_0/kT_{ps}^e$ versus $\tilde{T}_{ps}^i$ for the floating case ( ), following from the standard model [4] (curve a) and our model (curve b), for a hydrogen plasma and $\gamma^i = 3$, is shown in the Fig. 1. Our potential is more negative than the standard one, differing in magnitude by about 10%.

This discrepancy arises from the fact that our electron density is smaller than the Boltzmann one, so that the resulting space-charge density becomes more positive. From this argument we consider that our model should be better if the electrons in the sheath region are essentially collisionless. Hence, the question of the "correct" electron density distribution is a very important one, which should be considered in more depth in the future. Another point for further study is to improve the choice of the ion distribution functions at the sheath edge. While the cut-off Maxwellian used here seem to be a reasonable first choice, better fits could be obtained, e.g., from detailed inspection of existing kinetic plasma wall transition studies.

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